PCEM1 - ED2 - Statique des fluides - 2009-2010

Ex. 1 $\quad p = 1025$ mbar $= 102500$ Pa

Mercure : $h = \dfrac{p}{\rho g} = \dfrac{102500}{13546 \cdot 9{,}81} = 0{,}771$ m

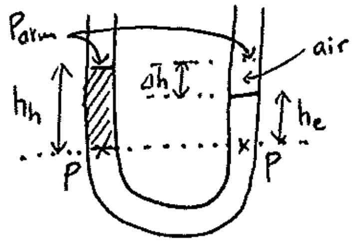
Eau : $h = \dfrac{p}{\rho g} = \dfrac{102500}{1000 \cdot 9{,}81} = 10{,}45$ m

Ex. 2 L'expérience est un succès si le poids de l'eau est inférieur à la force atmosphérique

$P_{eau} < F_{atm}$, $m_{eau} g < p_{atm} \cdot S$, $\rho_{eau} h_{eau} \cdot S \cdot g < p_{atm} \cdot S$, $h_{eau} < \dfrac{p_{atm}}{\rho_{eau} \cdot g} = 10{,}33$ m

La surface du verre ne joue aucun rôle

Ex. 3



$p_{atm} + \rho_h g h_h = p_{atm} + \rho_{air} g \Delta h + \rho_e g h_e$

$\rho_h h_h = \underbrace{\rho_{air} \Delta h}_{\text{négligeable}} + \rho_e h_e \qquad h_e = \dfrac{\rho_h}{\rho_e} h_h$

$h_{huile} = \dfrac{V_{huile}}{S} = \dfrac{47{,}1 \cdot 10^{-6}}{\left(\frac{\pi \cdot 0{,}02^2}{4}\right)} = 0{,}15$ m $\qquad h_e = 0{,}12$ m $\qquad \Delta h = 0{,}03$ m

Ex. 4 1) $V_{Hg} = \dfrac{m}{\rho_0} = \dfrac{3928{,}34}{13546} = 0{,}29$ m^3 $\qquad V = L \times h \times \ell = 1 \times 0{,}5 \times 0{,}5 = 0{,}25$ m^3 $\qquad V_{Hg} - V = 0{,}04$ m^3

$V_0 = \dfrac{0{,}04}{4} = 0{,}01$ m^3 $\qquad H_0 = \dfrac{V_0}{S} = \dfrac{0{,}01}{0{,}02} = 0{,}5$ m

2) $H_1 + h_1 = H_2 + h_2 = H_3 + h_3$

$H_1 + H_2 + H_3 + H_4 = 4 H_0$

$p_{atm} + \rho_0 g H_1 + \rho_1 g h_1 = p_{atm} + \rho_0 g H_2 + \rho_2 g h_2 = p_{atm} + \rho_0 g H_3 + \rho_3 g h_3 = p_{atm} + \rho_0 g H_4 + \rho_{air} g h_{air}$

$\rightarrow H_2 = H_1 + h_1 - h_2$, $H_3 = H_1 + h_1 - h_3$

$\rightarrow \rho_0 H_1 + \rho_1 h_1 = \rho_0 H_4 + \underbrace{\rho_{air} h_{air}}_{\text{négligeable}}$, $\quad H_4 = H_1 + \dfrac{\rho_1}{\rho_0} h_1$

$\rightarrow H_1 + H_1 + h_1 - h_2 + H_1 + h_1 - h_3 + H_1 + \dfrac{\rho_1}{\rho_0} h_1$, $\quad H_1 = H_0 - \dfrac{1}{4}\left[\left(2 + \dfrac{\rho_1}{\rho_0}\right) h_1 - h_2 - h_3\right]$

$= 495$ mm

$H_2 = 505$ mm, $H_3 = 475$ mm, $H_4 = 525$ mm

$\rightarrow \rho_0 H_2 + \rho_2 h_2 = \rho_0 H_4 + \underbrace{\rho_{air} h_{air}}_{\text{négligeable}}$, $\quad \rho_2 = \rho_0 \dfrac{(H_4 - H_2)}{h_2} = 681$ kg/m^3

$\rho_3 = \rho_0 \dfrac{(H_4 - H_3)}{h_3} = 1600$ kg/m^3

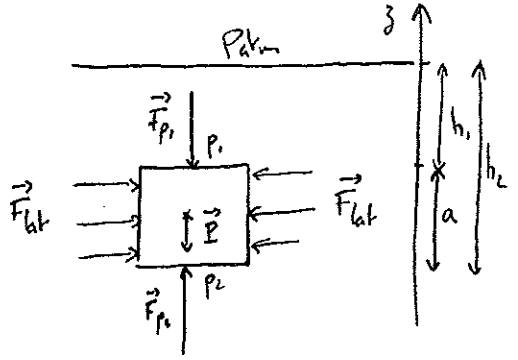
3) $H'_1 + H'_2 + H'_3 + H'_4 = 4 H_0$

$p_{atm} + \rho_0 g H'_1 + \rho_1 g h_1 = p_{atm} + \rho_0 g H'_2 + \rho_2 g h_2 = p_{atm} + \rho_0 g H'_3 + \rho_3 g h_3 = \rho_0 g H'_4$

$H'_1 = H'_4 - \dfrac{p_{atm}}{\rho_0 g} - \dfrac{\rho_1}{\rho_0} h_1$, $\quad H'_2 = H'_4 - \dfrac{p_{atm}}{\rho_0 g} - \dfrac{\rho_2}{\rho_0} h_2$, $\quad H'_3 = H'_4 - \dfrac{p_{atm}}{\rho_0 g} - \dfrac{\rho_3}{\rho_0} h_3$

$H'_4 = H_0 + \dfrac{3}{4} \dfrac{p_{atm}}{\rho_0 g} + \dfrac{1}{4}\left[\dfrac{\rho_1}{\rho_0} h_1 + \dfrac{\rho_2}{\rho_0} h_2 + \dfrac{\rho_3}{\rho_0} h_3\right]$

Ex. 5



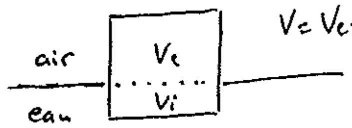
$$\vec F = \vec P + \vec F_{p_1} + \vec F_{p_2} + \underbrace{\sum \vec F_{lat}}_{\vec 0}$$

$$F_z = -\rho a^3 g - p_1 a^2 + p_2 a^2 = \underbrace{(p_2 - p_1)a^2}_{\text{Poussée d'Archimède}} - \underbrace{\rho g a^3}_{\text{Poids}}$$

$$\left.\begin{array}{l} p_2 = p_{atm} + \rho_{eau}\, g h_2 \\ p_1 = p_{atm} + \rho_{eau}\, g h_1 \end{array}\right\} \begin{array}{l} p_2 - p_1 = \rho_{eau}\, g (h_2 - h_1) \\ \quad = \rho_{eau}\, g\, a \end{array}$$

$$F_z = \rho_{eau}\, g a^3 - \rho g a^3 = +932\ N$$

Ex. 6



$V = V_e + V_i$

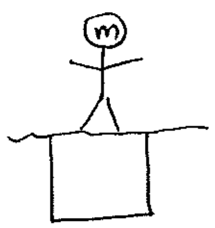
$$\rho g V = \rho_{eau}\, g V_i + \rho_{air}\, g V_e$$

$$\rho V = \rho_{eau} V_i + \rho_{air} V - \rho_{air} V_i$$

$$V_i = V \frac{\rho - \rho_{air}}{\rho_{eau} - \rho_{air}} = 29{,}877\ \ell$$

Si on néglige l'action de l'air, $V_i = V \frac{\rho}{\rho_{eau}} = 30\ \ell$

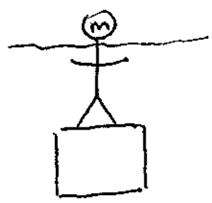
Ex. 7



La force à appliquer pour maintenir le cube sous l'eau est égale en norme à celle qui le fait remonter : $mg = 932\ N \rightarrow m = 95\ kg$

$mg + \rho g V = \rho_{eau}\, g V$, $m = V(\rho_{eau} - \rho) = 125\,(100 - 240) = 95\ kg$

Rq. : on a négligé, l'action de l'air (poussée d'Archimède) sur le personnage



$mg + \rho g V < \rho_{eau}\, g V + \rho_{eau}\, g V_{personnage}$ → l'ensemble remonte à la surface

Ex. 8 Tant que la montgolfière n'a pas atteint son altitude maximale, la poussée d'Archimède est supérieure en norme au poids du ballon et du gaz chaud.

$V_b = \frac{\pi d^3}{6}$, $mg + \rho_c g V_b = \rho_s (1 - \alpha h_{max}) V_b$ $\qquad \rho_s = 1{,}29\ kg/m^3$

$$h_{max} = \underbrace{\frac{(\rho_s - \rho_c)}{\alpha \rho_s}}_{2995\ m} - \underbrace{\frac{m}{\alpha \rho_s V_b}}_{1106\ m} = 1889\ m$$

Rq. : si $m = 270{,}8\ kg$, $h_{max} = 0$

Ex. 9 Le fruit flotte entre les deux fluides : $\rho V_0 g = \rho_1 x V_0 g + \rho_2 (1-x) V_0 g$

$$\rho = \rho_1 x + \rho - \rho_2 x$$

$$x = \frac{\rho - \rho_2}{\rho_1 - \rho_2} = \frac{850 - 800}{1000 - 800} = 0{,}25$$